**Solutions to Mid-Term Test**

1. (a) Write down the Hamiltonian of a particle of the mass moving in the plane subject to the potential energy .

(b) Given a wave function . Evaluate the normalization coefficient .

Thus .

1. (a) What is the Hermitian operator? Discuss the properties of a Hermitian operator.

For any two functions, if the following equation holds, then is called Hermitian operator.

Any eigenvalue of Hermitian operator is a real number.

Eigenfunctions of Hermitian operator with different eigenvalues are orthogonal.

(b) Prove that is a Hermitian operator.

Given any two functions and ,

Thus is a Hermitian operator.

Q.E.D.

1. What is the Heisenberg’s uncertainty principle for position and momentum? Why is it related to the commutator of the two operators?
2. For position and momentum along its direction, their uncertainties satisfy the following Heisenberg’s uncertainty principle

For position and momentum along other directions, we have

1. For two observables and , the product of their uncertainty is related to the expectation value of the commutator of the corresponding operators and .

Specify , , we have , thus

For , , we have , thus

1. An electron resides within a potential well of infinite depth and a width .
2. Write down its Schrödinger equation;

Denote the mass of electron as . Suppose it is confined in , then

where the potential is

The Schrödinger equation is

1. Confirm where is an integer is the solution of the above Schrödinger equation;

This wavefunction is the solution of above Schrödinger equation in .

It also satisfies boundary conditions , thus is indeed the solution of above Schrödinger equation.

1. Calculate the normalization coefficient .

Thus