Exercises

8C.1(a) The rotation of a molecule can be represented by the motion of a point mass moving over the surface of a sphere. Calculate the magnitude of its angular momentum when $l=1$ and the possible components of the angular momentum on an arbitrary axis. Express your results as multiples of $\hbar$.

8C.2(a) The wavefunction, $\psi(\phi)$, for the motion of a particle in a ring is of the form $\psi = Ne^{im\phi}$. Determine the normalization constant, N.

8C.3(a) Calculate the minimum excitation energy of a proton constrained to rotate in a circle of radius 100 pm around a fixed point.

8C.4(a) The moment of inertia of a CH_4 molecule is $5.27\times10^{-47}\,\mathrm{kg\,m^2}$. What is the minimum energy needed to start it rotating?

8C.5(a) Use the data in Exercise 8C.4(a) to calculate the energy needed to excite a CH_4 molecule from a state with $l=1$ to a state with $l=2$.

8C.6(a) What is the magnitude of the angular momentum of a CH_4 molecule when it is rotating with its minimum energy?

8C.7(a) Draw scale vector diagrams to represent the states (i) $l=1$, $m_l=+1$, (ii) $l=2$, $m_l=0$.

8C.8(a) The number of states corresponding to a given energy plays a crucial role in atomic structure and thermodynamic properties. Determine the degeneracy of a body rotating with $l=3$.

TOPIC 8C rotational motion

Discussion questions

8C.1 Discuss the physical origin of quantization of energy for a particle confined to motion around a ring.

8C.3 Describe the vector model of angular momentum in quantum mechanics. What features does it capture? What is its status as a model?

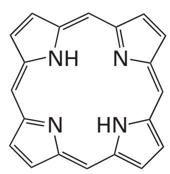
Integrated activities

8.4 Determine the values of $\Delta x = (\langle x^2\rangle - \langle x\rangle^2)^{1/2}$ and $\Delta p = (\langle p^2\rangle - \langle p\rangle^2)^{1/2}$ for the ground state of (a) a particle in a box of length L and (b) an harmonic oscillator. Discuss these quantities with reference to the uncertainty principle.

8.5 Repeat Problem 8.4 for (a) a particle in a box and (b) a harmonic oscillator in a general quantum state (n and v, respectively).

Problems

8C.1 The particle on a ring is a useful model for the motion of electrons around the porphine ring (2), the conjugated macrocycle that forms the structural basis of the haem group and the chlorophylls.



2 Porphine (porphin) ring

We may treat the group as a circular ring of radius 440 pm, with 22 electrons in the conjugated system moving along the perimeter of the ring. In the ground state of the molecule each state is occupied by two electrons. (a) Calculate the energy and angular momentum of an electron in the highest occupied level. (b) Calculate the frequency of radiation that can induce a transition between the highest occupied and lowest unoccupied levels.

8C.3 Evaluate the z-component of the angular momentum and the kinetic energy of a particle on a ring that is described by the (unnormalized) wavefunctions (a) $e^{i\phi}$, (b) $e^{-2i\phi}$, (c) $\cos\phi$, and (d) $(\cos\chi)e^{i\phi} + (\sin\chi)e^{-i\phi}$.

8C.5 Calculate the energies of the first four rotational levels of $^1H^{127}I$ free to rotate in three dimensions, using for its moment of inertia $I=\mu R^2$, with $\mu = m_H m_I/(m_H + m_I)$ and $R = 160$ pm.

8C.7 Confirm that $Y_{3,+3}$ is normalized to 1. (The integration required is over the surface of a sphere.)

8C.9 Develop an expression (in Cartesian coordinates) for the quantum mechanical operators for the three components of angular momentum starting from the classical definition of angular momentum, $\boldsymbol{l} = \boldsymbol{r}\times\boldsymbol{p}$. Show that any two of the components do not mutually commute, and find their commutator.

8C.11 Show that $[l^2, l_z] = 0$, and then, without further calculation, justify the remark that $[l^2, l_q] = 0$ for all $q = x, y$, and z.