Discussion questions

7B.1 Describe how a wavefunction summarizes the dynamical properties of a system and how those properties may be predicted.

7B.2 Discuss the relation between probability amplitude, probability density, and probability.

7B.4 What are the advantages of working with normalized wavefunctions?

7C.1 Suggest how the general shape of a wavefunction can be predicted without solving the Schrödinger equation explicitly.

7C.2 Describe the relationship between operators and observables in quantum mechanics.

7C.3 Account for the uncertainty relation between position and linear momentum in terms of the shape of the wavefunction.

Exercises

7B.1(a) Consider a time-independent wavefunction of a particle moving in three-dimensional space. Identify the variables upon which the wavefunction depends.

7B.1(b) Consider a time-dependent wavefunction of a particle moving in two-dimensional space. Identify the variables upon which the wavefunction depends.

7B.2(a) Consider a time-independent wavefunction of a hydrogen atom. Identify the variables upon which the wavefunction depends. Use spherical polar coordinates.

7B.4(a) For the system described in Exercise 7B.3(a), what is the probability of finding the light atom in the volume element $d\phi$ at $\phi=\pi$?

7B.4(b) For the system described in Exercise 7B.3(b), what is the probability of finding the electron in the range dx at $x=L/2$?

7B.5(a) For the system described in Exercise 7B.3(a), what is the probability of finding the light atom between $\phi=\pi/2$ and $\phi=3\pi/2$?

7B.5(b) For the system described in Exercise 7B.3(b), what is the probability of finding the electron between $x=L/4$ and $x=L/2$?

7C.1(b) Construct the potential energy operator of a particle subjected to a Coulomb potential.

7C.3(b) Functions of the form $\cos(n\pi x/L)$ can be used to model the wavefunctions of electrons in metals. Show that the wavefunctions $\cos(n\pi x/L)$ and $\cos(m\pi x/L)$, where $n\neq m$, are orthogonal for a particle confined to the region $0\leq x\leq L$.

7C.5(a) An electron in a carbon nanotube of length L is described by the wavefunction $\psi(x)=\sin(2\pi x/L)$. Compute the expectation value of the position of the electron.

7C.6(b) A light atom rotating around a heavy atom to which it is bonded is described by a wavefunction of the form $\psi(\phi)=e^{i\phi}$ with $0\leq\phi\leq2\pi$. If the operator corresponding to angular momentum is given by $(\hbar/i)d/d\phi$, compute the expectation value of the angular momentum of the light atom.

7C.7(a) Calculate the minimum uncertainty in the speed of a ball of mass 500 g that is known to be within 1.0 μm of a certain point on a bat. What is the minimum uncertainty in the position of a bullet of mass 5.0 g that is known to have a speed somewhere between 350.000 01 m s^{-1} and 350.000 00 m s^{-1}?

7C.7(b) An electron is confined to a linear region with a length of the same order as the diameter of an atom (about 100 pm). Calculate the minimum uncertainties in its position and speed.

7C.8(b) The speed of a certain electron is 995 km s^{-1}. If the uncertainty in its momentum is to be reduced to 0.0010 per cent, what uncertainty in its location must be tolerated?

Problems

7B.1 Normalize the following wavefunctions: (i) $\sin(n\pi x/L)$ in the range $0\leq x\leq L$, where $n=1, 2, 3, \ldots$ (this wavefunction can be used to describe delocalized electrons in a linear polyene), (ii) a constant in the range $-L\leq x\leq L$, (iii) $e^{-r/a}$ in three-dimensional space (this wavefunction can be used to describe the electron in the ion He^+), (iv) $xe^{-r/2a}$ in three-dimensional space. *Hint*: The volume element in three dimensions is $d\tau=r^2dr\sin\theta\,d\theta\,d\phi$, with $0\leq r<\infty$, $0\leq\theta\leq\pi$, $0\leq\phi\leq2\pi$.

7B.3 A particle free to move along one dimension x (with $0\leq x<\infty$) is described by the unnormalized wavefunction $\psi(x)=e^{-ax}$ with $a=2\,m^{-1}$. What is the probability of finding the particle at a distance $x\geq1$ m?

7B.7 Suppose that the state of the vibrating atom in Problem 7B.6 is described by the wavefunction $\psi(x)=Nxe^{-x^2/2a^2}$. Where is the most probable location of the particle?

7C.8 The normalized wavefunctions for a particle confined to move on a circle are $\psi(\phi)=(1/2\pi)^{1/2}e^{-im\phi}$, where $m=0, \pm1, \pm2, \pm3, \ldots$ and $0\leq\phi\leq2\pi$. Determine $\langle\phi\rangle$.

7C.9 A particle freely moving in one dimension x with $0\leq x<\infty$ is in a state described by the wavefunction $\psi(x)=a^{1/2}e^{-ax/2}$, where a is a constant. Determine the expectation value of the position operator.

7C.14 A particle is in a state described by the wavefunction $\psi(x)=(2a/\pi)^{1/4}e^{-ax^2}$, where a is a constant and $-\infty\leq x\leq\infty$. Verify that the value of the product $\Delta p\Delta x$ is consistent with the predictions from the uncertainty principle.

7C.15 A particle is in a state described by the wavefunction $\psi(x)=(2a)^{1/2}e^{-ax}$, where a is a constant and $0\leq x\leq\infty$. Determine the expectation value of the commutator of the position and momentum operators.