7B.3(a) An unnormalized wavefunction for a light atom rotating around a heavy atom to which it is bonded is $\psi(\phi)=e^{i\phi}$ with $0\le\phi\le 2\pi$. Normalize this wavefunction.

7B.3(b) An unnormalized wavefunction for an electron in a carbon nanotube of length L is $\sin(2\pi x/L)$. Normalize this wavefunction.

7B.4 The ground-state wavefunction for a particle confined to a one-dimensional box of length L is $\psi=(2/L)^{1/2}\sin(\pi x/L)$. Suppose the box is 10.0 nm long. Calculate the probability that the particle is (a) between $x=4.95$ nm and 5.05 nm, (b) between $x=1.95$ nm and 2.05 nm, (c) between $x=9.90$ nm and 10.00 nm, (d) in the right half of the box, (e) in the central third of the box.

7B.5 The ground-state wavefunction of a hydrogen atom is $\psi=(1/\pi a_0^3)^{1/2}e^{-r/a_0}$ where $a_0=53$ pm (the Bohr radius). (a) Calculate the probability that the electron will be found somewhere within a small sphere of radius 1.0 pm centred on the nucleus. (b) Now suppose that the same sphere is located at $r=a_0$. What is the probability that the electron is inside it?

7B.6 Atoms in a chemical bond vibrate around the equilibrium bond length. An atom undergoing vibrational motion is described by the wavefunction $\psi(x)=Ne^{-x^2/2a^2}$, where a is a constant and $-\infty<x<\infty$. (a) Normalize this function. (b) Calculate the probability of finding the particle in the range $-a\le x\le a$. *Hint*: The integral encountered in part (ii) is the error function. It is provided in most mathematical software packages.

7C.2(a) Confirm that the kinetic energy operator, $-(\hbar^2/2m)d^2/dx^2$, is hermitian.

7C.2(b) The operator corresponding to the angular momentum of a particle is $(\hbar/i)d/d\phi$, where ϕ is an angle. Is this operator hermitian?

7C.3(a) Functions of the form $\sin(n\pi x/L)$ can be used to model the wavefunctions of electrons in a carbon nanotube of length L. Show that the wavefunctions $\sin(n\pi x/L)$ and $\sin(m\pi x/L)$, where $n\neq m$, are orthogonal for a particle confined to the region $0\le x\le L$.

7C.4(a) A light atom rotating around a heavy atom to which it is bonded is described by a wavefunction of the form $\psi(\phi)=e^{im\phi}$ with $0\le\phi\le 2\pi$ and m an integer. Show that the $m=+1$ and $m=+2$ wavefunctions are orthogonal.

7C.5(b) An electron in a carbon nanotube of length L is described by the wavefunction $\psi(x)=(2/L)^{1/2}\sin(\pi x/L)$. Compute the expectation value of the kinetic energy of the electron.

7C.6(a) An electron in a one-dimensional metal of length L is described by the wavefunction $\psi(x)=\sin(\pi x/L)$. Compute the expectation value of the momentum of the electron.

7C.8(a) The speed of a certain proton is 0.45 Mm s^{-1}. If the uncertainty in its momentum is to be reduced to 0.0100 per cent, what uncertainty in its location must be tolerated?

7C.9(a) Determine the commutators of the operators (i) d/dx and $1/x$, (ii) d/dx and x^2.

7C.9(b) Determine the commutators of the operators a and a^+, where $a=(x+ip)/2^{1/2}$ and $a^+=(x-ip)/2^{1/2}$.

7C.1 Write the time-independent Schrödinger equations for (a) an electron moving in one dimension about a stationary proton and subjected to a Coulomb potential, (b) a free particle, (c) a particle subjected to a constant, uniform force.

7C.3 Identify which of the following functions are eigenfunctions of the operator d/dx: (a) e^{ikx}, (b) k, (c) kx, (d) e^{-ax^2}. Give the corresponding eigenvalue where appropriate.

7C.4 Determine which of the following functions are eigenfunctions of the inversion operator $\hat{i}$ which has the effect of making the replacement $x\rightarrow -x$): (a) x^3-kx, (b) $\cos kx$, (c) x^2+3x-1. State the eigenvalue of $\hat{i}$ when relevant.

7C.6 Show that the product of a hermitian operator with itself is also a hermitian operator.

7C.7 Calculate the average linear momentum of a particle described by the following wavefunctions: (a) e^{ikx}, (b) $\cos kx$, (c) e^{-ax^2}, where in each one x ranges from $-\infty$ to $+\infty$.

7C.10 The wavefunction of an electron in a linear accelerator is $\psi=(\cos\chi)e^{ikx}+(\sin\chi)e^{-ikx}$, where χ (chi) is a parameter. (i) What is the probability that the electron will be found with a linear momentum (a) $+k\hbar$, (b) $-k\hbar$? (c) What form would the wavefunction have if it were 90 per cent certain that the electron had linear momentum $+k\hbar$? (c) Evaluate the kinetic energy of the electron.

7C.12 The ground-state wavefunction of a hydrogen atom is $\psi=(1/\pi a_0^3)^{1/2}e^{-r/a_0}$. Calculate (a) the mean potential energy and (b) the mean kinetic energy of an electron in the ground state of a hydrogenic atom.

7C.13 Show that the expectation value of an operator that can be written as the square of an hermitian operator is positive.

7C.16 Evaluate the commutators (a) $[\hat{H},\hat{p}_x]$ and (b) $[\hat{H},\hat{x}]$ where $\hat{H}=\hat{p}_x^2/2m+\hat{V}(x)$. Choose (i) $V(x)=V$, a constant, (ii) $V(x)=\frac{1}{2}k_f x^2$.